Instructor's Commentary

The following pages present information useful to instructors for planning the presentation of the materials in the text. Commentary for each chapter includes the following information:

- The objective of the chapter, restated from the text
- Approximate amount of class time to spend on that section
- Suggested homework assignments
- Important terms and concepts
- Lesson notes, including teaching suggestions and references to appropriate Exploration(s) for that section
- Additional class examples
- Supplemental resource activities
- Problem notes, giving highlights and background for most problems
- Advanced Placement problems that you may wish to use when reviewing this chapter

1 Limits, Derivatives, Integrals, and Integrals

Often in previous courses, students have heard that "you will need this when you get to calculus," and when they finally arrive at calculus, they expect to see the power, beauty, and usefulness of mathematics in the real world. Typically, their first day of class begins a weeks-long review of all precalculus mathematics, and what should be the beginning of the culmination of their high school mathematics education starts off like most of the other math courses they have taken. Some tune out, others get discouraged when they find that they don't remember everything quite like they think they should, and the calculus class can begin to bog down.

This text, however, *starts off* with calculus. Review of previous mathematics courses occurs where it is needed, not at the beginning. Calculus classes are made more exciting for both student *and* instructor when students begin to work with some of the major concepts of calculus from the very *first* day of class. In the first week students will have seen examples of limit, derivative, and definite integral. This way of presenting the concepts is made time-efficient by use of the graphing calculator.

1-1 The Concept of Instantaneous Rate

Objective

Given the equation for a function relating two variables, estimate the instantaneous rate of change of the dependent variable with respect to the independent variable at a given point.

Class Time

AB and BC Courses: 1 day

Suggested Homework Assignment

AB and BC Courses: Problems 1 and 2

Important Terms and Concepts

Function

Independent variable

Dependent variable

Limit

Average rate

Instantaneous rate of change

Derivative

Lesson Notes

The text example introduces the concepts of limit and derivative via analysis of the motion of a door connected to an automatic closer. The example provides a relatively simple real-world situation in which one variable, the angle of the door, depends on another variable, time. If your classroom door has an automatic closer, you might have students demonstrate the motion. The average rate has units degrees per second. Because the variables have different units, students see more clearly what the rate, and thus the derivative, means. The function $d = 200t \cdot 2^{-t}$ involves the product of a monomial and an exponential, and is thus complicated enough that students who have had some exposure to derivatives in previous courses will probably not be able to differentiate it algebraically. Discussion of the concepts of limit and local linearity arises naturally when students use values of t nearer and nearer to 1 to estimate the instantaneous rate of the change.

The idea that a function can be defined in a restricted domain is brought out by having the door slam shut after 7 seconds.

Exploration 1-1a in the supplementary materials allows you to present the door-closer problem as either a directed class activity or a cooperative group activity. Review of the concepts of function, dependent and independent variables, and average rate of change will arise naturally as students work this problem. If you do this as a cooperative exercise, allow time at the end of the period for some sort of summary activity so that students will be sure they have learned the right things.

The Technology Project *Instantaneous Rate,* provided in the *Instructor's Resource Book,* covers the same material as Exploration 1-1a, but is enriched with use of The Geometer's Sketchpad. You may wish to use the first four or five Technology Projects to cover the first chapter of the text using The Geometer's Sketchpad.

Consider saving discussion of the mechanics of the class (how grades are computed, other policies, and so on) to a later time, or committing them to a handout that students can read outside of class. Then you can start the year off with calculus!

Problem Notes

The problems in this section illustrate the "a, b, c, . . ." format in which students do many mathematical things with the same problem, instead of the same one thing with many problems. Many problems have **names,** which help you and the students tell at a glance what the problem is about. The names also provide a way for you to locate a problem you may recall from earlier in the course by looking it up in the **Index of Problem Titles** in the text.

- *Problem 1,* the Pendulum Problem, appears again later in the text in connection with calculus of trig functions and again with parametric functions.
- *Problem 2,* the Board Price Problem, illustrates the fact that the instantaneous rate, price per foot, for a board may decrease for a while, then increase for very long boards.

1-2 Rate of Change by Equation, Graph, or Table

Objective

Given a function $y = f(x)$ specified by a graph, by a table of values, or by an equation, describe whether the y-value is increasing or decreasing as x increases through a particular value, and estimate the instantaneous rate of change of y at that value of x.

Class Time

AB and BC Courses: 1 day

Suggested Homework Assignment

AB and BC Courses: All Q problems, and Problems 11–29 odd

Important Terms and Concepts

Particular equation

General equation

Meaning of derivative

Verbal definition of limit

Lesson Notes

After discussion of the previous day's work, have the students sketch various graphs, and discuss what type of function is represented: linear, polynomial, power, and so on. Next have them determine whether the function is increasing, decreasing, or not changing at $x = 1$. How do they decide if the function is increasing or decreasing at a particular point? (If the graph is rising as the x-values increase, the function is increasing. If the graph is falling as the x-values increase, the function is decreasing.) If the function is increasing or decreasing, tell whether the rate of change is slow or fast. How do they decide whether a rate of change is slow or fast? (If the graph has a large amount of vertical change, shown by steepness greater than 45°, for a small amount of horizontal change, it is changing rapidly. If there is relatively little vertical change for a relatively large horizontal change, shown by a gentle change, the function is changing slowly. Discussion of Example 1 in the text and similar problems will help students make these generalizations.)

You may wish to note that the text gives the form $y = mx + b$ as the general equation for a linear function, but students may be more familiar with $y = a + bx$ or $y = ax + b$.

Problems 2–10 even can be used for a three-minute oral classroom exercise on the connection between the graph and the rate of change.

Problem 12, similar to Example 2 in the text, can be used as a five-minute classroom exercise on the connection between the slope of the tangent line to the graph of a function $y = f(x)$ at $x = c$ and the derivative of $f(x)$ at $x = c$. *Problem 14,* similar to Example 3, can also be used as another short classroom exercise introducing students to the concept of the derivative as a limit of a difference quotient.

Exploration 1-2a can be reproduced and used as a ten-minute review of the various function types in addition to reinforcing the connection between the graph and the rate of change. Students who take time to use their graphers instead of just pencil and paper will find that they have not been time-efficient!

It is suggested that students read through the text examples on their own.

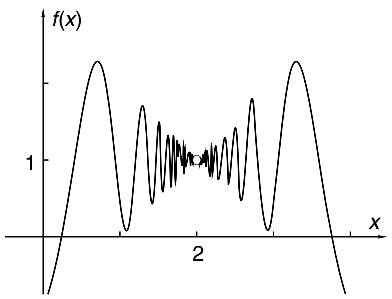
Now students are ready for a statement of the meaning of derivative. You might want to start making some connections between the relationship of the curve at a particular point and what is happening to its derivative at that point. For example, using the quadratic $f(x) = x^2 + 2x - 2$, they could estimate the derivative at $x = 1$ and relate it to the fact that the function is increasing quickly there. At $x = -2$ in the same quadratic, the function is decreasing slowly.

A verbal definition of a limit is given at the end of this section. Students should see a connection between the definition and the average rates in this section. Note that although the widely accepted terminology states that $f(x)$ *approaches* L, the fact is that $f(x)$ *stays close* to L when x is kept close to c (but not equal to c). For instance,

$$f(x) = 1 + (x - 2)\sin\left(\frac{10}{x - 2}\right)$$

approaches 1 as x approaches zero. But, as shown in Figure IC 1-2a (on the next page), the graph moves away from and toward the limit an infinite number of times as x approaches zero.

Figure IC 1-2a



When discussing the meaning of derivative, make sure that students distinguish between a *geometric* tangent line to a circle's graph and a *calculus* line tangent to the graph at a point. The calculus tangent line is the best linear approximation to the curve at a point, if such an approximation exists. The tangent line may cross the function's graph at the point of tangency, or cross at other places, unlike a geometric tangent line to a circle.

Supplemental Resource Activities

A Watched Cup Never Cools. Lab Activity 3: What Goes Down, Must Come Up . . . , p. 11. In this lab students use a motion sensor to measure the movement of a bouncing ball. They then analyze rates of change numerically to develop a better understanding of velocity and acceleration.

Problem Notes

- *Q* problems ("Quickie" problems, if you choose to call them that!) make their first appearance in this section. Most problem sets will begin with ten short problems. Some of these problems are intended for review of skills from previous sections, chapters, or courses. Others are to test general knowledge. Speed, not detailed work, is the key to the *Q* problems. Students should be able to finish all ten in less than five minutes, as they are expected to do on standardized tests and in mathematics contests. As well as ongoing, spiraled review, these problems could be used as short, timed quizzes.
- *Problems 1–10* are similar to Example l. These problems require students to estimate the derivative at a given point for a function given graphically. They also give a review of the *types* of function (polynomial, exponential, rational, and so on). These problems can be done orally in class or as group activities.
- *Problems 11–16* are similar to Examples 2 and 3. In Problems 11 and 12, students practice estimating the derivative graphically. In Problems 13–16, students are led to estimate the derivative numerically, and they begin to look at limits of difference quotients. Remind students to express their derivative estimates using appropriate units. Problems 15 and 16 refer to graphing in a "friendly window." This refers to a grapher window for which a particular x-value (usually an integer) falls on a pixel, allowing you to see holes and asymptotes more clearly. To find such a window quickly on a TI grapher, use the zoom decimal feature. You can adjust this window by multiplying the minimum and maximum window values by a scale factor.
- *Problems 17 and 18* give practice in estimating the derivative at a point for a function given numerically (by tabular data).
- *Problems 19–28* are similar to those in Exploration 1-2. Students approximate the derivative at a particular point for a function given algebraically (by equation).
- *Problems 29 and 30* have the students write their understanding of the meaning of derivative and limit, supporting their ideas with numerical and graphical methods.

1-3 One Type of Integral of a Function

Objective

Given the equation or the graph of a function, estimate on a graph the definite integral of the function between $x = a$ and $x = b$ by counting squares.

Class Time

AB and BC Courses: 1 day

Suggested Homework Assignment

AB and BC Courses: All *Q* problems, Problems 1–11 odd, and 12–14 all

Important Terms and Concepts

Definite integral

Meaning of definite integral

Lesson Notes

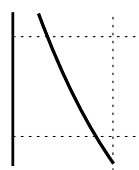
You may want to begin class with this situation. Suppose a person can jog a mile in 16 minutes. How far has the person gone after jogging 32 minutes? (2 miles) Sketch a graph of this constant velocity function. What are the variables to be used? (Let t = the time in minutes spent jogging and $v(t) = 1/16$, the speed of the jogger in mi/min.) Ask students to find a geometric shape to shade that represents the distance traveled. They should sketch a rectangle that is 32 units horizontally and 1/16 units vertically, and it's an easy step for them to see that the area of this rectangle is the same as the distance jogged. Because students are accustomed to thinking of area as being measured in square units, some discussion of the appropriate units may be necessary. In this example, the area of the rectangle is given by (1 mi/16 min)(32 min) = 2 mi, because the minute units cancel.

Exploration 1-3a can be used as an additional or alternative classroom example. In this exploration, students investigate the motion of a car as it slows down after having passed a truck on the highway. The product (dependent)(independent) has the units (ft/s)(s) and thus represents feet that the car has gone. The second situation on that sheet involves the cross section of a football at various distances from one end. The product (dependent)(independent) has units (square inches)(inches) and thus represents the cubic inches in the volume of the football. Students seem surprised and pleased when they reach this conclusion!

A common error students make is to ignore the measurement of each side of a "square" and assume that each has an area of 1×1, so caution your students. As time permits, you may want the class to work unassigned even problems together in their groups.

The counting of partial squares on the graph should be done to the nearest 0.1 unit. Don't allow the students to use fractions because that would imply precise results rather than approximate. The following thought process shows how students can reach a decision quickly.

Figure IC 1-3a



- The region under the graph in Figure IC 1-3a is more than half a square, so the only possibilities are 0.6, 0.7, 0.8, or 0.9.
- The region is not *much* more than half, so 0.6 is reasonable.

As long as the estimated area of each square is no more than ± 0.1 unit from the actual value, the estimate to the integral will be reasonably accurate.

The Technology Project *One Type of Integral* covers the topics of this lesson using The Geometer's Sketchpad.

Problem Notes

- *Problems 1–4* are similar to Example 1. These are skill-building problems in which students plot an accurate graph and count the squares. One or two of these are enough to assign.
- *Problems 5 and 6* are similar to Example 2, where the graph is already plotted. Note that in Problem 5 the region is from $t = 5$ to 25, not from $t = 0$ to 25.

- *Problems 7 and 8* give a review of estimating the derivative of a given function.
- *Problems 9 and 10* involve integrals and derivatives. In each problem the equation for a velocity function is given and students find both distance (integral) and acceleration (derivative).
- *Problem 11* introduces students to the fact that a definite integral can be negative if the region lies below the x-axis.
- *Problems 12–14* reinforce the memory of the meanings of derivative and definite integral and the verbal definition of limit.

1-4 Definite Integrals by Trapezoids, from Equations and Data

Objective

Estimate the value of a definite integral by dividing the region under the graph into trapezoids.

Class Time

AB and BC Courses: 1 day

Suggested Homework Assignment

AB and BC Courses: All *Q* problems, Problems 1–5 odd, 6, and Problems 7–13 odd

Important Terms and Concepts

Trapezoidal rule

Lesson Notes

Counting squares to estimate a definite integral is both tedious and time-consuming. A more efficient method is to divide the region under a graph into vertical strips and connect the boundaries to form trapezoids. The sides of the trapezoids give a better approximation to the curve, and it's easy to compute the area of a trapezoid. As well, counting squares requires an accurate graph of a function, whereas trapezoids can be used for a function presented algebraically (by equation) or numerically (in tabular form).

Students need to recall the formula for computing the area of a trapezoid, which is easily remembered by taking the product of the height and the *average* of the bases, or $\frac{1}{2}(b_1 + b_2)h$.

Exploration 1-4a, the Rocket Problem, may be used as a small-group discovery exercise or as the basis for a guided class discovery. It takes about 25 minutes of class time if you simply download a program for the trapezoidal rule similar to the one in the *Instructor's Resource Book and CD-ROM*. Students accept the validity of the program because it gives the same answer they got "by hand" using four trapezoids.

The Technology Project *Rectangular and Trapezoidal Accumulation* covers the topics of this lesson using The Geometer's Sketchpad.

Supplemental Resource Activities

Exploring Calculus with The Geometer's Sketchpad. Going the Distance, p. 22. In this activity, students explore how area on a time-velocity graph represents distance, and they examine change in position by estimating area with trapezoids.

Problem Notes

- *Problems 1 and 2* are similar to Example 1. They give the equation of a function and have the students use the trapezoidal rule to approximate area.
- *Problems 3 and 4* are similar to Example 2. They give function data in table form and have the students use the trapezoidal rule to approximate area. Problem 3 refers to the late Navy Lieutenant Kara Hultgreen, the author's former student who was killed while attempting to land her F-14 on a carrier off the coast of California in early 1995.
- *Problems 5 and 6* have the students write programs to compute approximations for the definite integral using the trapezoidal rule. To save time, you can download a program similar to the one in the *Instructor's Resource Book and CD-ROM*. This program will be needed for several later problems in this Problem Set.
- *Problems 7 and 8* provide practice for students to verify the accuracy of their trapezoidal rule programs.
- *Problems 9 and 10* apply integrals to real-world problems.
- *Problems 11–13* help students consolidate their knowledge about integrals and the error involved in approximating the definite integral using the trapezoidal rule.

1-5 Calculus Journal

Objective

Start writing a journal in which you can record things you've learned about calculus and what questions you still have about certain concepts. In doing so, you'll gain practice in writing about mathematics, and you'll have a source of reference in your own words to review before tests.

Class Time

AB and BC Courses: 0 to 1/2 day

Suggested Homework Assignment

AB and BC Courses: Problem 1, either prior to their test on Chapter 1 or as an assignment due the day after their first test

Lesson Notes

Students are perhaps familiar with journals from their language classes, but the idea of a *mathematics* journal may seem foreign. Throughout the text, guided journal prompts occur and should be used by students to help them accomplish the objectives of the course.

If the journal is assigned before the first test, make time to have students discuss their journal entries before the test. It's a good idea to go over the text for this section verbatim with the class to help form the foundation for insightful journal entries throughout the year. After all, they are getting ready to take their first test in calculus, and since the approach taken thus far hasn't been particularly formulaic, some students may be feeling a bit uneasy because they can't figure that they will have "5 problems of type a, 6 of type b, and so on" on the test. The format suggested by the text, using a bound notebook or spiral-bound notebook with large index cards for pages, is particularly good and easy for students to use for continued reference.

If you assign the journal after the first test, students can record their feelings about the test and about what they still need to learn. Thinking critically and questioning thoughtfully throughout the remainder of the year will help prepare them for their future education and careers.

1-6 Chapter Review and Test

Objective

Review and practice the major concepts of calculus presented in the chapter: derivatives and integrals.

Class Time

AB and BC Courses: 2 days (including 1 for testing)

Suggested Homework Assignment

AB and BC Courses

Day 1: Problems R1–R5 in preparation for the test

Day 2 (after Chapter 1 Test): *Either* Problems C1 and C2; *or* Problem C3; *or* Journal, Problem Set 1-5, Problem 1; *or* Problem Set 2-1, Problems 1–3 all

Lesson Notes

If students have started their journals before the Chapter 1 test, it can be helpful on the day before the first test to have them share with the class some of the things they have written. The results should yield good review questions. You might also pick up some ideas for the test if you haven't already written it!

The Chapter Test included in the text is divided into two parts: calculator inactive and calculator active. Because the Advanced Placement Calculus Exam is divided in the same manner, class assessments should model this situation to allow students sufficient practice prior to the exam's administration in May. You may wish to have the students work the Chapter Test (or one of the tests in the *Instructor's Resource Book*) in class as a **rehearsal.** The name "rehearsal" seems to bring students' minds into focus better than the name "review" or "worksheet." They may work individually (a "dress rehearsal!") or in groups. By working the Chapter Test in class, students will have access to the answers, which do not appear in the Selected Answers section of the text. The Review Problems are more appropriate for work at home because the answers do appear in the text.

Problem Notes

As pointed out in the text, the Review Problems are numbered according to the preceding sections of the chapter. The Concept Problems allow the students to apply their knowledge to new situations.

- *Problem R1,* the Bungee Problem, may puzzle students who think the sinusoidal equation applies from the instant Lee jumps. The domain does not start until the bungee starts becoming taut, at $t = 3$ seconds. Lee jumped from higher than the 170 feet that forms the upper bound for the sinusoid. Later on students will investigate sinusoids with an amplitude that decreases with time, as the bungee will do in the real world.
- *Problem C1,* the Exact Value of a Derivative Problem, gives students a preliminary glimpse of the reasoning they will do in Chapter 3 to find algebraic formulas for derivatives.
- *Problem C2,* the Tangent to a Graph Problem, is a "do-it-yourself" introduction to a technique used throughout the text to check a numerical value of a derivative graphically. If the line with slope equal to the derivative is really tangent to the graph, then the derivative is (probably) correct.
- *Problem C3* can be used as the basis for your classroom presentation of the formal definition of limit, or as a group discovery exercise, if not assigned as homework after the test. In this problem, students see a removable discontinuity in the graph at $x = 3$, with a limit of 5 as x approaches 3. After simplification, students find that $f(x) = 4x - 7$, provided $x \neq 3$. Students are then led to find values for delta, given a specific value for epsilon, and to find a value for delta in terms of epsilon, for any value of epsilon.
- *Problems TI–T8* are designed to be worked without the use of a graphing calculator, while *Problems T9–T18* presume the use of a graphing calculator.